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The Roles of Patterning and Spatial Skills in Early Mathematics Development

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Abstract

Because math knowledge begins to develop at a young age to varying degrees, it is important to identify foundational cognitive and academic skills that might contribute to its development. The current study focused on two important, but often overlooked skills that recent evidence suggests are important contributors to early math development: patterning and spatial skills. We assessed preschool children's repeating patterning skills, spatial skills, general cognitive skills and math knowledge at the beginning of the pre-kindergarten year. We re-assessed their math knowledge near the end of the school year, with complete data for 73 children. Children's repeating patterning and spatial skills were related and were each unique predictors of children's math knowledge at the same time point and seven months later. Further, repeating patterning skills predicted later math knowledge even after controlling for prior math knowledge. Thus, although repeating patterning and spatial skills are related, repeating patterning skills are a unique predictor of math knowledge and growth. Both theories of early math development and early math standards should be expanded to incorporate a role for repeating patterning and spatial skills.

Keywords: Mathematics development; patterning skills; spatial skills; cognitive skills; preschool

Highlights:

- Preschool children's repeating patterning and spatial skills were related
- Patterning and spatial skills predicted math knowledge at beginning and end of prek
- Theories and standards for early math should include pattern and spatial skills

The Roles of Patterning and Spatial Skills in Early Mathematics Development

Proficiency in mathematics is important for academic, economic, and life success. For example, greater academic achievement in math is related to college completion, higher earnings, and better health decisions (Adelman, 2006; Reyna, Nelson, Han, & Dieckmann, 2009; Ritchie & Bates, 2013). Individual differences in math knowledge emerge in preschool and are fairly stable. For example, general math knowledge in the final year of preschool (pre-kindergarten year, or prek) and kindergarten predict math achievement across primary and secondary school (Duncan et al., 2007; Jordan, Kaplan, Ramineni, & Locuniak, 2009; Nguyen et al., 2016; Watts, Duncan, Siegler, & Davis-Kean, 2014). Further, weak math knowledge at school entry among low-income children largely explains their weak math knowledge later in elementary school (Jordan et al., 2009). Such findings have led to increased attention to math instruction in preschool. For example, recent observations in U.S. preschools indicate that math accounted for 25% of instructional time (Piasta, Pelatti, & Lynnine, 2014), which is substantially more time than the 14% of instructional time spent on math in first-grade classrooms in the 1990's (NICHD Early Child Care Research Network, 2002).

Because math knowledge begins to develop at a young age to varying degrees, it is imperative that we identify foundational cognitive and academic skills that contribute to this development and explain its variation. The goal of the current study was to focus on two important, but often overlooked, skills that recent evidence suggests are important contributors to early math development: repeating patterning and spatial skills. We begin with background on current math education standards, which give minimal attention to the role of each skill in early math development. Then, we review empirical evidence for the importance of spatial and repeating patterning skills for math knowledge, considering each in turn. Finally, we explore

how spatial and repeating patterning skills may be related to each other.

Math Education Standards

The Common Core State Standards (2010), or local variations of them, are currently being implemented in schools across the country. However, these standards give minimal attention to patterning or spatial skills in the early grades. Patterning skills encompass the ability to notice and use predicable sequences, such as a predictable array of shapes or sounds or functional relations between two variables (Burgoyne, Witteveen, Tolan, Malone, & Hulme, 2017). With young children, the focus is on skills with repeating patterns (i.e., linear patterns that have a unit that repeats, such as circle-circle-square-circle-circle-square). Thus, the current paper focuses on repeating patterning skills.

The importance of patterning skills for math achievement is currently under debate. Previously, patterning skills were included in national and state math standards for prek and kindergarten under the algebra strand (National Association for the Education of Young Children, 2014; National Council of Teachers of Mathematics, 2006). However, in 2008, the National Mathematics Advisory Panel (2008) concluded: "In the Major Topics of School Algebra set forth in this report, patterns are not a topic of major importance. The prominence given to patterns in PreK–8 is not supported by comparative analyses of curricula or mathematical considerations" (p. 59). The only evidence cited in the report was a curriculum analysis indicating that only one of the six highest performing countries on an international assessment emphasized patterns in the early grades (Schmidt & Houang, 2007). In part, the recommendation of the panel reflects the paucity of evidence that existed at the time on the value of patterning skills. The Common Core State Standards (2010) followed this recommendation, not including patterns as a math content standard at any grade level. Anecdotal evidence suggests that teachers may not welcome this

change. One kindergarten teacher recently complained to a colleague: "They took patterns away from us! Kids actually liked to do patterns."

Spatial skills also receive minimal attention in the Common Core State Standards (2010) for math. Spatial skills encompass cognitive skills related to visual imagery and mental manipulation of spatial information (Uttal et al., 2013). Such skills are only mentioned once in the early grades, as part of a content standard for Kindergarten. Spatial skills also received minimal attention in past national and state math standards for prek and kindergarten (National Association for the Education of Young Children, 2014; National Council of Teachers of Mathematics, 2006). However, there is a growing advocacy for more attention to spatial skills from early in education (Newcombe, 2010; Verdine, Golinkoff, Hirsh-Pasek, & Newcombe, 2014a). Thus, research on the potential roles of spatial and patterning skills for early math knowledge development is urgently needed. Next, we review past research on each skill and its relations to math knowledge.

Spatial Skills

First consider spatial skills. Young children regularly engage their spatial skills as they play with blocks, puzzles, and videogames (Jirout & Newcombe, 2015; Levine, Ratliff, Huttenlocher, & Cannon, 2012; Newcombe, 2010; Verdine et al., 2014a). In one large, representative sample, 75% of 4- to 7-year old children were reported to play with blocks, puzzles and board games at least sometimes (i.e., three to five times per week), and 25% were reported to play with these toys often (i.e., six times or more per week) (Jirout & Newcombe, 2015). Further, children in this study who played with these toys more often also had higher spatial skills. Some young children also hear many spatial words when talking with their parents, and frequency of hearing spatial words is also linked to spatial skills (Pruden, Levine, &

Huttenlocher, 2011).

Spatial skills are core cognitive skills such as spatial visualization (the ability to imagine and mentally transform spatial information), form perception (the ability to copy and distinguish shapes from other shapes, including symbols), and visual-spatial working memory (i.e., *visual-spatial WM,* the ability to hold the locations of different objects, landmarks etc. in working memory) (Mix & Cheng, 2012; Mix et al., 2016; Uttal et al., 2013). Spatial skills are present in infancy and continue to develop through childhood (Newcombe, 2010; Uttal et al., 2013). For example, children's ability to represent and transform spatial information improves with age (Levine, Huttenlocher, Taylor, & Langrock, 1999), and their visual-spatial WM capacity increases over development as well (Isaacs & Vargha-Khadem, 1989; Li & Geary, 2013).

Links Between Spatial Skills and Math Knowledge

How might spatial skills influence math knowledge? The most common theoretical perspective is that mathematical thinking is supported by spatial representations (Mix & Cheng, 2012). From mental number lines to geometric figures, information about locations in space are often processed when solving math problems. For example, some people generate schematic representations of math problems that include the spatial relations described in the problems, and these people are more likely to solve the problems correctly (Hegarty & Kozhevnikov, 1999). Further, visual-spatial WM is needed to solve some types of math problems, such as addition problems that involve carrying (Caviola, Mammarella, Cornoldi, & Lucangeli, 2012). Additionally, mathematical ideas are often grounded in experiences moving through space, such as moving objects to combine or separate sets (addition and subtraction), and stepping along a path as an experience supporting the link from a number line to magnitudes (Griffin, 2004). Although most theory and research is with school-age children and adults, one theory of early

math development has incorporated spatial skill, specifically visual-spatial WM, as a foundational cognitive skill for supporting early numeracy knowledge, which subsequently supports later math achievement (LeFevre et al., 2010).

There is robust evidence that spatial skills are linked to individual differences in math knowledge. Children and adults with better spatial skills relative to their peers also have better math skills (see Mix & Cheng, 2012 for a review). For example, visual-spatial WM is predictive of math knowledge in school-age children (Bull, Espy, & Wiebe, 2008; Meyer, Salimpoor, Wu, Geary, & Menon, 2010; Raghubar, Barnes, & Hecht, 2010), as is form perception (Lachance & Mazzocco, 2006; Zhang & Lin, 2015) and spatial visualization (Guay & McDaniel, 1977; Kyttälä, Aunio, Lehto, Van Luit, & Hautamäki, 2003). A few of these studies have confirmed a link between spatial skills and later math knowledge, controlling for earlier math knowledge or examining growth in math knowledge (Bull et al., 2008; Lachance & Mazzocco, 2006). Although most longitudinal evidence comes from school-age children and adults, recent evidence indicates this association is present before school entry. Spatial assembly skill at age 3 was predictive of math knowledge concurrently and two years later, controlling for earlier math knowledge and executive function skills (Verdine, Golinkoff, Hirsh-Pasek, & Newcombe, 2017), and visual-spatial WM at age 4.5 predicted math achievement through Grade 3, controlling for reading achievement and executive function skills (Bull et al., 2008). Further, spatial skill in infancy may be predictive of math knowledge at age 4 (Lauer & Lourenco, 2016). After reviewing the literature, Mix and Cheng (2012) concluded: “the connection between space and math may be one of the most robust and well-established findings in cognitive psychology” (p. 198).

Despite the robust longitudinal relations, causal evidence that improving spatial skills

leads to improvements in math knowledge is mixed, especially for children. Three studies have reported that improving children's spatial skills also improved their performance on math assessments (Cheng & Mix, 2013; Hawes, Moss, Caswell, Naqvi, & MacKinnon, 2017; Lowrie, Logan, & Ramful, 2017). However, two other studies failed to find an effect (Hawes, Moss, Caswell, & Poliszczuk, 2015; Xu & LeFevre, 2016), and there are no consistent differences across studies that do and do not find an effect.

Overall, we need to better understand the relation between spatial skills and math knowledge in children, especially before children begin kindergarten. We selected measures of three spatial skills that have been shown to predict math knowledge in past research with preschool or school age children, as reviewed above: visual-spatial WM, spatial visualization and form perception. Individual studies often included only one spatial measure, making it impossible to make predictions about the relative contributions of the different spatial skills. In addition, confirmatory factor analyses suggest that a broad range of spatial measures all load onto a single factor in school-age children (Mix et al., 2016). Thus, we focus on the contributions of the set of spatial skills for predicting math knowledge in preschool first. We then explore potential unique contributions of each spatial skill, over and above the other spatial skills.

Repeating Patterning Skills

Young children, teachers, and parents all regularly work with repeating patterns. For example, some children spontaneously create patterns during their free play at preschool (Ginsburg, Inoue, & Seo, 1999; Ginsburg, Lin, Ness, & Seo, 2003). Further, U.S. preschool teachers often view patterning activities as important (Clarke, Clarke, & Cheeseman, 2006; Economopoulos, 1998). In one small study, U.S. teachers reported engaging their students in frequent repeating patterning activities, and parents reported doing a range of pattern activities

with their children at home at least once a week (Rittle-Johnson, Fyfe, Loehr, & Miller, 2015).

Young children's repeating patterning skills become systematically more sophisticated in preschool and kindergarten (Clements & Sarama, 2009; Papic, Mulligan, & Mitchelmore, 2011; Rittle-Johnson, Fyfe, McLean, & McEldoon, 2013; Starkey, Klein, & Wakeley, 2004). Children first learn to work with simple alternating AB patterns such as a red-blue pattern, and then learn to identify patterns with three and four item units (e.g., ABB and AABB patterns). By the end of preschool, many children are able to *complete* (identifying the missing item in a pattern), *duplicate* (make an exact replica of a model pattern) and *extend* (continue an existing pattern by at least one unit of repeat) repeating patterns (Clements & Sarama, 2009; Papic et al., 2011; Rittle-Johnson et al., 2015). Some are even able to *abstract repeating patterns*—recreating a model pattern using a different set of materials.

Despite evidence for how children of particular ages do on a particular task, there are very few available assessments focused on children's repeating patterning skills. We used the only measure with evidence for its reliability and validity with preschool children (Rittle-Johnson et al., 2013) and developed and tested a new measure of children's repeating patterning skills. Our goal was to create a new measure using tasks commonly used by teachers in preschool and kindergarten classrooms, including tasks that were easier than those included on the existing measure (e.g., completing patterns and working with AB patterns). Thus, this new measure captured earlier developing patterning skills and was more aligned with common classroom activities than the existing research-based patterning assessment. It also provided a second measure of a central construct in this research and a measure that can easily be created and implemented by teachers.

Links Between Repeating Patterning Skills and Math Knowledge

How might repeating patterning skills support math knowledge? Identifying, extending, and describing predictable sequences (patterns) in objects and numbers are core to mathematical thinking (Charles, 2005; Sarama & Clements, 2004; Steen, 1988). Both counting and arithmetic principles describe generalizations of predictable sequences. For example, the next number name in the count sequence represents a magnitude that is exactly one more than the previous number name (i.e., the successor function). Working with repeating patterns provides early opportunities to identify and describe predictable sequences, without requiring numeracy knowledge. This is why some early math education researchers consider patterning to be central to early math thinking (Papic et al., 2011; Sarama & Clements, 2004; Warren & Cooper, 2006).

Recent longitudinal and causal evidence indicates that patterning skills are important to math development. First, consider longitudinal evidence. Repeating patterning skill at the end of prekindergarten was a unique predictor of fifth- and sixth-grade math achievement, after controlling for a wide variety of other math and cognitive skills, including measures of early numeracy knowledge (Nguyen et al., 2016; Rittle-Johnson, Fyfe, Hofer, & Farran, 2016). Further, repeating patterning skill at the end of preschool was equally predictive of different fifth-grade math topics, including numeration, algebra, and geometry (Rittle-Johnson et al., 2016).

Evidence from other studies suggests the relation may be causal. Instruction on repeating patterns supports future knowledge of growing patterns (Papic et al., 2011) and ratios (Warren & Cooper, 2007), and instruction on several types of patterns, including repeating patterns, supports general math achievement (Kidd et al., 2013; Kidd et al., 2014). For example, Kidd and colleagues found that struggling first-grade students who were randomly assigned to receive supplemental pattern instruction in the classroom throughout the school year had greater math

achievement at the end of the school year than children who received one of several alternative interventions (Kidd et al., 2013; Kidd et al., 2014). Indeed, the patterning intervention was as effective and sometimes more effective than a more general math intervention that reviewed an eclectic set of math topics.

Potential Relations Between Spatial and Repeating Patterning Skills

Although both spatial and repeating patterning skills play an important role in math development, they have been studied independently in past research. Thus, little is known about the relations between the two types of skills. For example, children may rely on spatial skills to complete repeating patterning tasks, especially when the tasks include working with visual patterns constructed with objects. The spatial skill of form perception is likely needed to distinguish and match objects that make up patterns, and visual-spatial WM may be needed to hold and manipulate the spatial information in WM. In the only study including measures of spatial and patterning skills, preschool children's visual-spatial WM was moderately correlated with their repeating patterning skill, more so than their verbal short-term memory or inhibitory control (Collins & Laski, 2015).

Given the potential overlap between repeating patterning and spatial skills, research is needed to tease apart the specific relations among and between each skill and math knowledge. In particular, repeating patterning skills could be related to math knowledge because of their relation to spatial skills, rather than having a direct effect on math knowledge. In other words, it could be that spatial skill is the unaccounted-for third variable in past research linking patterning skills and math knowledge.

In summary, both repeating patterning and spatial skills are thought to support math knowledge. Correlational evidence indicates that both repeating patterning and spatial skills are

predictive of math knowledge, both concurrently and years later (Mix & Cheng, 2012; Rittle-Johnson et al., 2016). Intervention research suggests that pattern instruction, and perhaps spatial instruction, can improve math knowledge (Cheng & Mix, 2013; Kidd et al., 2013; Kidd et al., 2014; Papic et al., 2011). Though patterning and spatial skills share overlapping characteristics, almost no work has studied them together, despite their similarities. It could be that spatial skills are driving the relation between repeating patterning skills and math knowledge. Further, it is important to determine these predictive relations after controlling for other general cognitive skills that influence math performance. Both verbal ability and verbal working-memory capacity are related to early math performance (Bull et al., 2008; Duncan et al., 2007), so were included as control variables in the current study.

Research Goals and Hypotheses

The goal of this research was to explore the cross-domain development of repeating patterning skills, spatial skills, and math knowledge in prekindergarten. We focused on prekindergarten children (ages 4 and 5) because some patterning and spatial skills are in place by age 4, and individual differences in these skills in prekindergarten are already predictive of later math knowledge (Rittle-Johnson et al., 2013; Rittle-Johnson et al., 2016; Verdine et al., 2017). Further, much less is known about the relations between spatial skills and math knowledge before children enter Kindergarten. We tested four hypotheses.

1. **Hypothesis 1**: Repeating patterning skills and spatial skills (i.e., visual-spatial WM, form perception and spatial visualization) will be moderately correlated with one another.
2. **Hypothesis 2**: Our set of spatial skills will predict math knowledge, both concurrently and seven-months later, over and above the effects of general cognitive skills (age, verbal ability, and verbal WM).

3. **Hypothesis 3**: Repeating patterning skills will predict math knowledge, both concurrently and seven-months later, over and above the effects of spatial skills and general cognitive skills (age, verbal ability, and verbal WM).
4. **Hypothesis 4**: Spatial and repeating patterning skills will predict later math knowledge, after controlling for prior math knowledge, using the same math measure at both time points. This hypothesis was more tentative. Because performance on a particular measure is often best predicted by previous performance on the measure, this is a very rigorous test. For studies on the link from spatial skills to math knowledge, only a few have controlled for prior math knowledge, and we only identified two studies that controlled for prior math knowledge using the same assessment as both a predictor and an outcome (Bull et al., 2008; Lachance & Mazzocco, 2006). Prior longitudinal research on patterning skills and later math knowledge have controlled for prior math knowledge using different math measures.

Theoretically, this research contributes to a more comprehensive theory of early academic development in math by considering the role of spatial and repeating patterning skills. Practically, the timing of this research is critical to inform efforts to revise and implement the Common Core State Standards (2010), which give little attention to patterning and spatial skills.

Method

Participants

Initial participants were 79 children who were recruited from six preschool programs: three public, one Head Start center, and two private. Two children would not assent to participate in the study, one child was withdrawn from the study because of other commitments, and three children were no longer attending the participating preschools at Time 2 and could not be located. In the final sample of 73 children (57.5% females), children were an average age of 4

years 7 months (*SD* = 4 months; range = 4 years 0 months – 5 years 2 months) when first assessed. As shown in Table 1, our sample was ethnically and economically diverse.

Procedure

Within the first quarter of the school year, children were individually assessed at their preschools in two 30-minute sessions by one of 5 assessors, approximately five days apart. In some cases, children required a third session to complete all the assessments (*n* = 6). The first session included assessments of verbal ability, research-based patterning, form perception, and spatial visualization. In the second session, children completed assessments of visual-spatial and verbal WM, math knowledge, and teacher-based patterning.

At Time 2, during the final quarter of the school year, children completed the math and verbal STM and WM assessments in a single 20-minute session. The average delay to Time 2 was 6.8 months (*SD* = 7.6 days). During a second session, children completed additional assessments that were beyond the scope of the current study.

Measures

Repeating patterning skills.

Research-based patterning assessment. This assessment measured preschoolers' ability to duplicate, extend, abstract, and identify units of repeating visual patterns, and consisted of nine items at four levels of difficulty, described and validated in previous studies (Miller, Rittle-Johnson, Loehr, & Fyfe, 2016; Rittle-Johnson et al., 2013; Rittle-Johnson et al., 2015). One of the abstract pattern items on the previous version of the assessment was converted to a duplicate pattern item to make the assessment easier. To reduce testing time and child frustration, a stop criteria was implemented, such that the assessment was stopped once children answered all extend items or all abstract items incorrectly. The assessment took up to 15 minutes to

administer, and the assessment showed good internal consistency in our sample (Cronbach's α = .84). However, an unexpectedly large number of children did not solve any pattern items correctly (*n* = 26, 36% of sample), limiting variability on this assessment. As in past studies, we generated ability estimates for children using a Rasch model with a Laplace approximation and empirical Bayesian prediction method that has been shown to be stable for sample sizes around 50 (Cho & Rabe-Hesketh, 2011). Laplace approximation was implemented in R (http://www.r-project.org), using the glmer function of the lme4 package (Bates, Maechler, & Dai, 2008).

Teacher-based patterning assessment. The new patterning assessment was developed using pre-existing patterning worksheets found on websites with resources for early-childhood educators (for examples, see http://www.nuttinbutpreschool.com/mm-patterning-center-worksheets/). This 10-item assessment was intended to measure children's repeating patterning ability in a manner similar to what might be used in a classroom setting, including easier tasks than given on the research-based patterning assessment. Children were presented with pictures of model patterns and given a set of small, laminated pictures to complete the patterning task. The pattern units were more variable than on the research-based patterning assessment, and included AB, ABB, ABC and AABB pattern units. See Figure 1 for sample items for the 4 types of tasks. The first two were easier pattern completion tasks, and were the most commonly found tasks in our search: (a) what comes next and (b) missing item. The third was extending patterns. The fourth task type was only found on one website, but was included because it was potentially interesting - matching patterns. See Table 2 for a list of items, including the complexity of the pattern unit as well as item-level statistics. The assessment took approximately six minutes to administer. Children earned a point for each item answered correctly, and internal consistency was good (Cronbach's α = .83). Children's performance was indexed using ability estimates

generated using a Rasch model with a Laplace approximation method.

Spatial skills.

Form perception. The Position in Space subtest of the *Developmental Test of Visual Perception–Second Edition* (Hammill, Pearson, & Voress, 1993) was chosen as a measure of form perception, as used by Lachance and Mazzocco (2006). This 25-item test requires children to identify an image from a set of four or more figures that match a target image on the right, with a specified stop criteria. The assessment took less than five minutes to administer. Children earned a point for each item answered correctly, and according to the manual, internal consistency is high (Cronbach's $\alpha > .80$) for children ages 4 through 10 years.

Spatial visualization. Block Design, a subtest of the *Wechsler Preschool and Primary Scale of Intelligence–Fourth Edition* (Wechsler, 2012) was used to measure spatial visualization. In this 17-item test, children were shown either a picture or a model of a block structure and were asked to recreate it using red and white colored blocks. The assessment was administered according to standardized instructions, and took approximately nine minutes to administer. According to the manual, internal consistency is high (Cronbach's $\alpha = .85$) for children ages 4 through 7.

Visual-spatial working memory. The Corsi Block Tapping Task was used as a measure of visual-spatial WM. Children completed the task using the *PathSpan* program, which is a version of the Corsi Block adapted for use on an iPad and with young children (available at https://hume.ca/ix/pathspan.html) and used in previous research (LeFevre et al., 2010; Xu & LeFevre, 2016). The display showed nine green circles in fixed positions, which were presented to the children as lily pads. A frog "jumped" from lily pad to lily pad, and children were asked to "copy the frog's path by touching the same lily pads." First, the experimenter demonstrated the

task on one trial and children practiced on two trials, with feedback. The number of lily pads that children needed to remember began with two and increased to a maximum of eight lily pads, with two trials for each span length. Testing was discontinued if children answered two trials of the same span length incorrectly; to improve chances that children had adequate opportunity to understand the task, they were given up to two additional two-span trials if they failed the first two trials. After children completed the forward span task, they were given new instructions to copy the lily pads backwards (i.e., "touch the lily pads in the opposite way the frog did") and two practice trials with feedback. The task took approximately seven minutes to complete. Both the forward and backwards order are thought to similarly recruit WM resources, and several studies have demonstrated no significant differences in performance on the forwards and backwards orders on the Corsi task (Isaacs & Vargha-Khadem, 1989; Kessels, van den Berg, Ruis, & Brands, 2008). Thus, children's scores on the Corsi task were calculated as the number of trials correct on both the forward and backward order of the task. An unexpectedly high number of children did not solve any trials correctly (n = 18, 25% of sample), limiting variability on this task. Past research has not reported evidence for the reliability of the measure, but internal consistency within our sample was fairly weak, with a Cronbach's alpha of .61.

Math knowledge. The REMA Short-Form contains a subset of items from the *Research-Based Early Mathematics Assessment* (Weiland et al., 2012). The test is comprised of 19 items, split into two sections: 13 items assessing children's numeracy knowledge, and 6 items assessing their geometric knowledge. Items for each section were ordered by IRT difficulty estimates from the norming sample, and we implemented the stop criteria validated by the authors (stop after 3 incorrect answers in a row on each section). The assessment took about 12 minutes to administer. Items were scored according to criteria specified by the authors, although scores on

3 of the 4 polytomous items had to be collapsed into fewer categories to accommodate the distribution of scores in our sample. Weiland and colleagues (2012) reported a Cronbach's α of .71 and .79 in two samples, and internal consistency in our sample was similar (.75 at T1, .77 at T2). Considering the two sections separately, internal consistency was acceptable for the numeracy section (.75 at T1 and .76 at T2), but unacceptable for the geometric section, especially at Time 2 (.56 at T1 and .39 at T2).

IRT ability estimates were generated using a partial credit model. To improve the precision of ability estimates for our sample size below 100, we used Empirical Bayes estimation to constrain the item parameters (Baker & Kim, 2004), using WinBUGS 1.4.3 (Spiegelhalter, Thomas, Best, & Lunn, 2003). The informative prior distribution on the item difficulty parameters and the sum-to-zero constraints on the item location and threshold parameters were chosen based on results reported in Weiland et al. (2012).

Verbal ability. The Picture Vocabulary Test from version 1.6 of the *NIH Toolbox app* assessed children's receptive vocabulary. Prior research has demonstrated good concurrent and discriminant validity in a diverse sample of 3- to 15-year-olds (Weintraub et al., 2013). This computer-adaptive test was administered to children on an iPad, and took approximately four minutes to complete. Age-corrected and standardized scores were used[1]. Vocabulary scores were used as a control for verbal ability, as vocabulary is strongly correlated with general IQ scores (Sattler, 2008).

Verbal short-term and working memory. At Time 1, a backward letter span task was administered (Ramirez, Chang, Maloney, Levine, & Beilock, 2016) based on the backwards digit

[1] One child's score was implausible (scored 5 standard deviations below the mean, even though child was a native English speaker who had normal conversation with experimenter and scored in typical range on all other measures), so we replaced this child's verbal score with the sample mean.

span task from the *Wechsler Intelligence Scale for Children*. Unfortunately, a majority of children in our sample struggled to understand "backwards" despite extensive training, with 60% failing to complete a single trial correctly (n = 47) and a mean of 0.87 trials correct. Because of extreme floor effects on this assessment, it was dropped from further analysis.

Because of this difficulty, we administered the forward and backward digit span task from the *Wechsler Intelligence Scale for Children* at Time 2 (Wechsler, 2003). The task began with a training phase to make the task more accessible to young children (see Miller, Rittle-Johnson, Loehr, & Fyfe, 2015) and the task took less than five minutes to complete. Forward digit span is typically used to assess verbal short-term memory (STM), and we administered it to use as a potential control variable given floor effects on the working memory measure at Time 1. The backward digit span is used to assess verbal WM. Children were given scores for the forward and backward orders, which were calculated by summing the number of total trials answered correctly for each order. A substantial number of children were unable to complete any backward trials correctly (n = 29, 40% of sample), again limiting variability on this assessment, but internal consistency in our sample was moderate (Cronbach's α = .71). Scores on the forward order task were often not correlated with performance on other measures, including scores on the backward order task ($r(71) = .104$, $p = .383$), so the forward order task was not included as a control variable in our models. This is in line with other research that verbal short-term memory is not a strong or consistent predictor of math achievement (Bull et al., 2008).

Data Preparation and Screening

All measures were screened for skew and kurtosis. One measure, visual-spatial WM had extreme skewness and kurtosis, driven by one child who scored over 4 standard deviations above

the mean, and 2 standard deviations above the next highest score. This child often scored 2 standard deviations above the mean on other measures. Visual inspection of scatterplots confirmed that she/he was biasing some correlations, so this student was dropped from analyses.

We tested for non-independence in math scores that might arise from children being nested within different schools, controlling for general cognitive skills, but intra-class correlations were near 0 at Time 1 and Time 2, indicating that OLS regression analyses were appropriate. We tested for multicollinearity by estimating variance inflation factors (VIF) for all independent variables, and all VIF scores for independent variables were < 2.74, indicating multicollinearity was not biasing the results. We controlled for three indicators of general cognitive skills in all analyses - age, verbal ability, and verbal WM. Finally, preliminary analyses indicated that none of the demographic factors (e.g., sex, financial assistance) were significant predictors of patterning, spatial, or math skills after controlling for age, verbal ability, and verbal WM. Thus, they were not included in the final models. We also considered dropping the three assessments on which a substantial number of children failed to get any trials correct (research-patterning, visual-spatial WM & verbal WM), but the assessments were consistently and moderately correlated with other assessments, indicating that it added value to include the assessments.

Results

Relations Among Variables

Descriptive statistics and correlations among key variables at both time points are presented in Table 3, with raw correlations presented above the diagonal. To provide a sense of the global relation between repeating patterning and spatial skills, we created composite scores for each (by averaging standardized scores on relevant measures) and include those composite

variables in the table. Significant positive correlations were found among all tasks. In most cases, these positive relations between target variables held after controlling for general cognitive skills (age, verbal ability, and verbal WM; partial correlations are presented below the diagonal in Table 3).

As predicted in Hypothesis 1, the patterning and spatial composite measures were strongly related, $r(70) = .59$, $p < .001$, and the relation was moderate after controlling for general cognitive skills, $pr(67) = .37$, $p = .002$. Partial cross-domain correlations among individual patterning and spatial measures ranged from .14 to .33, and several partial correlations were not significant. The partial correlations between the teacher-based patterning measure and individual spatial measures were small and not significant. In contrast, the research-based pattern assessment was more consistently related to individual spatial skills. Overall, patterning and spatial skills were moderately related, although not all individual measures were related.

In line with Hypotheses 2 and 3, the patterning and spatial composite measures were each moderately correlated with math knowledge at both Time 1 and Time 2, after controlling for general cognitive skills (ranging from .38 to .46). Partial correlations among individual patterning and spatial tasks with math knowledge at Time 1 and Time 2 ranged from .18 to .45 (see Table 3, below the diagonal). Teacher-based patterning, spatial visualization and form perception were significantly related to math knowledge at both time points; research-based patterning was only related to math knowledge at Time 1 and visual-spatial WM was only related to math knowledge at Time 2. Overall, patterning and spatial skills were each moderately correlated with math knowledge concurrently and over time.

Predictors of Math Knowledge

In order to determine the concurrent and predictive relations between spatial and

patterning skills on math performance, hierarchical linear regression analyses were performed with math knowledge at Time 1 and Time 2 serving as the dependent variables. The first regression block included age at the time of the math assessment, verbal ability, and verbal WM to control for general cognitive skills. The three spatial measures were then entered into the second block to test Hypothesis 2. Then, the two patterning measures were entered into the third block to test Hypothesis 3. To test Hypothesis 4, we conducted a third analysis, with math knowledge at Time 2 as the dependent variable and math knowledge at Time 1 included in the first regression block. Results are summarized in Table 4.

In line with Hypothesis 2, adding the three spatial measures in Block 2 resulted in significant improvement in model fit for predicting math knowledge at Time 1, $\Delta R^2 = .09$, $p = .012$, and at Time 2, $\Delta R^2 = .10$, $p = .008$, see Table 4 columns 1-9. However, individual spatial measures tended not to be unique predictors of math knowledge over and above other spatial measures and general cognitive skills. At Time 1, form perception was a somewhat reliable and unique predictor, $\beta = .20$, $p = .06$, and at Time 2, visual-spatial WM was a somewhat reliable and unique predictor, $\beta = .22$, $p = .07$. Floor effects and weak internal consistency on our visual-spatial WM measure suggest that this study may underestimate the true effects of visual-spatial WM. Overall, the set of spatial skills predicted math knowledge, but individual spatial skills tended not to be unique predictors.

In line with Hypothesis 3, including the two patterning measures in Block 3 resulted in significant improvement in model fit for predicting math knowledge at Time 1, $\Delta R^2 = .07$, $p = .007$, and at Time 2, $\Delta R^2 = .09$, $p = .002$, see Table 4. Considering individual patterning measures, the teacher-based patterning measure was a unique predictor, over and above all other measures, of math knowledge at Time 1, $\beta = .30$, $p = .007$, and at Time 2, $\beta = .40$, $p = .001$. The

researcher-based patterning measure was not a unique predictor over and above other measures, although floor effects on this measure limited variability, and thus its predictive potential.

In the third model, we evaluated Hypothesis 4 by adding math knowledge at Time 1 to the model in the first block to evaluate the impact of patterning and spatial skills in predicting improvement in math knowledge. After controlling for math knowledge at Time 1, including the three spatial measures in Block 2 did not result in significant improvement in model fit for predicting math knowledge at Time 2, $\Delta R^2 = .03$, $p = .157$, see Table 4, columns 10-13. Considering individual spatial measures for predicting math knowledge at Time 2, visual-spatial WM was a somewhat reliable and unique predictor over and above the other measures, $\beta = .18$, $p = .08$, but the other measures were not. Including the two patterning measures in Block 3 did result in significant improvement in model fit for predicting math knowledge at Time 2, $\Delta R^2 = .04$, $p = .036$. Considering individual patterning measures, the teacher-based patterning measure was a unique predictor, over and above all of the other measures, $\beta = .28$, $p = .012$.

Finally, we explored whether these findings held when predicting performance on only the numeracy section of the math measure. Most early math measures only include numeracy knowledge, so we explored whether our findings would likely generalize to other measures. Full results are presented in Table S1 in the supplemental materials. Support for Hypothesis 2 was mixed. The set of spatial skills did not predict Time 1 numeracy knowledge, $\Delta R^2 = .05$, $p = .109$, but it did predict Time 2 numeracy knowledge $\Delta R^2 = .08$, $p = .023$. Among the individual spatial skills, spatial visualization, and not visual-spatial WM, was a reliable predictor of Time 2 numeracy knowledge, $\beta = .23$, $p = .02$. After controlling for Time 1 numeracy knowledge, the set of spatial skills was a marginal predictor of numeracy knowledge at Time 2, $\Delta R^2 = .06$, $p = .065$, providing some support for Hypothesis 4.

Support for Hypothesis 3 was also mixed, but stronger. The set of patterning skills did not predict Time 1 numeracy knowledge, $\Delta R^2 = .04$, $p = .103$, but the teacher-based patterning measure was a marginal predictor on its own, $\beta = .22$, $p = .06$. The set of patterning skills did predict Time 2 numeracy knowledge $\Delta R^2 = .06$, $p = .014$, and the teacher-based patterning measure was a robust predictor, $\beta = .33$, $p < .01$. In line with Hypothesis 4, after controlling for Time 1 numeracy knowledge, the set of patterning skills continued to predict numeracy knowledge at Time 2, $\Delta R^2 = .05$, $p = .032$.

Discussion

The current study highlights the contribution of two important, but often overlooked, skills in early math development: spatial and repeating patterning skills. The two skill sets were related, and each set of skills was a unique predictor of math knowledge concurrently and 7-months later. We discuss the development of a new patterning skills measure, the potential relations between repeating patterning and spatial skills, and the relations between each of these skills and math knowledge. We end with educational implications of the findings, especially in light of recommendations to reduce attention to patterning in school math.

Measuring Repeating Patterning Skills

An important contribution of this study was the development and validation of a second measure of repeating patterning skills. Without evidence for the reliability and validity of measures, we cannot assess the validity of the inferences drawn from studies. Our goal was to create a new measure drawing on tasks commonly used by teachers in preschool and kindergarten classrooms, including tasks that capture earlier developing patterning skills and tasks that are more aligned with common classroom activities. Our new teacher-based patterning assessment shows great promise as a reliable and valid measure of patterning skills in preschool.

It had strong internal consistency, good face and construct validity, and evidence for convergent and predictive validity. Importantly, it was the most consistent predictor of math knowledge of all of our assessments. In addition, it can easily be created and implemented *by* teachers. This is imperative if teachers are to use assessments to appropriately plan lessons and measure student learning.

As expected, children were most successful at tasks involving AB patterns. In addition, pattern completion tasks, including deciding what comes next and what item is missing, were easier than pattern extension tasks (Clements & Sarama, 2009). The matching items were more difficult to interpret. The easiest item was matching an AB pattern using new materials, suggesting that this matching task is not tapping children's patterning abstraction skills as one might expect. On the researcher-based patterning assessment, children's responses must be at least 8 inches from the model pattern to reduce use of one-to-one object matching, as recommended by Clements and Sarama (2009). Further, some children respond with AB patterns, regardless of the model pattern (Rittle-Johnson et al., 2013), so a correct response on the AB matching item might not indicate abstraction of a pattern unit. We need a better understanding of what the pattern matching items might be tapping.

Overall, it is encouraging that patterning items often used by teachers show such promise for predicting future math knowledge. However, the more difficult items on our assessment were typically found in materials targeted towards kindergarten and first-grade students, highlighting that some preschool children may develop more advanced patterning skills than commonly recognized by teachers.

Relations Between Repeating Patterning and Spatial Skills

In line with Hypothesis 1, repeating patterning and spatial skills were related. There was

a strong relation between the two composite measures, which was maintained, although more moderate, after controlling for other general cognitive skills. Each of the individual assessments was also moderately correlated with one another. Matching objects (i.e., form perception) and detecting spatial relations between objects are both spatial skills that should support identifying repeating patterns made from objects, and these processes likely require visual-spatial WM resources.

Over and above general cognitive skills, the research-based patterning assessment was significantly correlated with individual spatial assessments; however, the teacher-based patterning assessment was not. Other research suggests that advanced patterning tasks like abstracting patterns and identifying the unit of repeat may be particularly demanding on visual-spatial WM (Collins & Laski, 2015), and only the research-based patterning assessment included these tasks. More advanced patterning tasks may also benefit from spatial visualization skills as well. In addition, perhaps other demands, such as ignoring more distractor response options on the teacher-based assessment, overshadowed the role of spatial skills on this measure. In general, patterning tasks may vary in their demands on spatial skills over and above other general cognitive skills.

Further, the relation between the two skills could be bi-directional. Indeed, some spatial tasks involve noticing patterns. For example, our spatial visualization assessment, Block Design, requires noticing patterns in visual information (e.g., arranging bi-colored blocks to duplicate a modeled design). The form perception assessment sometimes involved noticing the sequencing of individual shapes within a larger shape. Thus, patterning skills for noticing and using predictable sequences could be useful for completing spatial tasks. We need additional research to better understand the associations between patterning and spatial skills, including using other

assessments of patterning and spatial skills to better understand their relations. At the same time, the modest partial correlations, after controlling for other cognitive skills, suggest that the two skills are separate and distinct skills in preschool.

Spatial Skills and Math Knowledge

In line with Hypothesis 2, our set of spatial skills (i.e., form perception, spatial visualization and visual-spatial working memory) predicted math knowledge concurrently and seven-months later, over and above the effect of age, verbal ability and verbal WM. The math assessment used in the current study, the REMA-Brief, broadly assessed early math knowledge, including numeracy and geometry topics. Further, the set of spatial skills predicted numeracy knowledge seven-months later, indicating that the relation between spatial skills and math knowledge was not driven by the geometry items on the math assessment. Thus, past findings of links between particular spatial skills and early number knowledge before school entry (Bull et al., 2008; Lauer & Lourenco, 2016; Verdine et al., 2017) generalize to a broader array of spatial skills and a broader math knowledge measure.

However, as explored in Hypothesis 4, the set of spatial skills did not predict broad math knowledge seven months later over and above prior math knowledge, using the same math assessment at both time points. When considering only the numeracy subscale, the set of spatial skills was marginally related to later numeracy knowledge. Given how few studies have controlled for prior math knowledge when exploring links between spatial and math skills in children, we need more research on whether and when spatial skills predict *improvements* in math knowledge. According to some theories, spatial skills are important to mathematics because people use their spatial skills to solve math problems (Caviola et al., 2012; McKenzie, Bull, & Gray, 2003). This suggests that the relevant component of spatial skills for mathematics

knowledge will be captured by prior math knowledge, and thus spatial skills may not have much of a direct relationship with later math knowledge after controlling for prior math knowledge. However, a few studies have found added predictive value for particular spatial skills in early childhood (Bull et al., 2008; Lachance & Mazzocco, 2006; Verdine et al., 2017), and we suspect the answer will depend on the target math content assessed at each time point.

Further, individual spatial measures were inconsistently related to math knowledge, over and above the impact of the other spatial skills. This finding is inline with recent results that a broad range of spatial skills all load onto a single factor in school-age children (Mix et al., 2016). Most research on the links between spatial skill and math knowledge in childhood only measure a single spatial skill, often using a single measure. Little is known about the distinctions between spatial skills in preschool and whether they are separable constructs. We did not have an adequate sample size to investigate this issue, so future research is needed.

Nevertheless, visual-spatial WM may be one distinct spatial skill that predicts future math knowledge in preschool children. Although the predictive value of visual-spatial WM in the full model was only marginal in our study, floor effects and weak internal consistency on our visual-spatial WM measure suggest that our study may underestimate the true effects of visual-spatial WM. Indeed, Bull and colleagues (2008) found that 4-year-olds' visual-spatial WM predicted math knowledge in kindergarten, first grade and fifth grade. Similarly, visual-spatial WM in kindergarten was uniquely related to a range of math outcomes two years later, including numeration, calculation, and geometry knowledge (LeFevre et al., 2010). In part, visual-spatial WM is needed to solve some types of math problems, such as addition problems that involve carrying or mental arithmetic problems (Caviola et al., 2012; McKenzie et al., 2003). LeFevre and colleagues suggest that visual-spatial WM supports early numeracy knowledge, which

subsequently supports later math achievement (LeFevre et al., 2010).

Spatial visualization skill and form perception were not unique predictors of math knowledge in this study, although exploratory analyses suggest spatial visualization was a unique predictor of future numeracy knowledge. Better spatial visualization skill seems to help elementary school children adopt more sophisticated arithmetic strategies (Laski et al., 2013) and have more precise representations of numerical magnitude (Gunderson, Ramirez, Beilock, & Levine, 2012). We know little about the role of spatial visualization in earlier math thinking and learning, but it may play a similar role in early numeracy development. Even less is known about the role of form perception in math thinking and learning and whether it is a distinct spatial skill in young children.

Repeating Patterning Skills and Math Knowledge

In line with Hypothesis 3, repeating patterning skills in preschool predicted math knowledge, both concurrently and seven-months later, over and above the effects of spatial skills, age, verbal ability and verbal WM. Further, in line with Hypothesis 4, the predictive value of patterning skills held even after controlling for prior math knowledge, using the same math measure at both time points. Performance on a particular measure is often best predicted by previous performance on the measure, so this is a very rigorous test and had not been demonstrated in prior longitudinal research linking patterning skills to later math knowledge. Further, patterning skills were unique predictors of numeracy knowledge in particular, indicating that the relation between patterning skill and math knowledge was not driven by the geometry items on the math assessment.

Given the overlap between patterning and spatial skills, spatial skills could have been an unaccounted-for third variable in past research linking patterning skills and math knowledge.

The current evidence indicates that this is not the case. Even though spatial skills are related to patterning skills, repeating patterning skills were unique predictors of mathematics knowledge over and above the impact of spatial skills as well as other general cognitive skills.

The predictive role of repeating patterning skills for math knowledge in this study converges with past longitudinal and experimental research. Repeating patterning skills at the end of prek predicted math achievement in first and fifth grade (Rittle-Johnson et al., 2016). In particular, repeating patterning skills in preschool predicted symbolic mapping, calculation, and patterning knowledge in early elementary school, which in turn supported general math achievement in the middle grades (Rittle-Johnson et al., 2016). Further, improving repeating patterning skills in preschool supported the development of more advanced counting, symbolic mapping (e.g., mapping quantities to verbal number names) and calculation knowledge by the following school year, compared to children who had not received a patterning intervention in preschool (Papic et al., 2011). The current study further highlights the predictive role of repeating patterning skills for numeracy knowledge in particular, as well as for more general math knowledge.

Why might repeating patterning skill predict current and future math knowledge? Current evidence suggests that repeating patterning skills are linked to improved *numeracy* knowledge in early elementary school. Patterning skills involve deducing underlying rules in the sequence of objects, and numeracy knowledge also requires deducing underlying rules from examples, such as the successor principle for symbol-quantity mappings (e.g., the next number name means adding one). Repeating patterning skills may also promote some counting skills, such as counting by 2's or 5's. Because repeating patterning tasks do not require prior number knowledge, even preschool children can deduce underlying rules in the patterns. Developing

such skills with repeating patterns at a young age may support their noticing and use of patterns in numbers as they acquire basic numeracy knowledge.

Finally, the current findings highlight the importance of considering repeating patterning and spatial skills in theories of math development. A majority of research and theory on early math development focuses on numeracy knowledge (knowledge of the meaning of whole numbers and number relations). However, early math knowledge extends beyond knowledge of numbers (National Research Council, 2009). LeFevre and colleagues Pathways to Mathematics model includes one spatial skill – visual-spatial WM – as a cognitive precursor skills for promoting early numeracy performance (LeFevre et al., 2010). Rittle-Johnson and colleagues Early Math Trajectories Models includes repeating patterning skills as one of three early math skills that promotes later mathematics achievement (Rittle-Johnson et al., 2016). In general, more comprehensive theories of early math development are needed which consider a range of math topics and supporting skills.

Educational Implications

The current findings on the potential role of repeating patterning and spatial skills for math knowledge development are urgently needed. First, the current findings, in combination with other recent longitudinal and intervention research (Burgoyne et al., 2017; Kidd et al., 2013; Kidd et al., 2014; Papic et al., 2011; Rittle-Johnson et al., 2016), suggest that repeating patterning should be included in early math standards, contrary to the current Common Core State Standards (CCSS, 2010). The CCSS (2010) do include the mathematical practice "look for and make use of structure," and repeating patterning tasks may be useful for engaging this practice at an early age, a possibility not mentioned in the standards. In addition, evidence suggests that repeating patterning skills supports numeracy knowledge, so attention to repeating

patterning should not detract from numeracy development. Rather, repeating patterning tasks may be an additional method for promoting numeracy development.

There are several ways to scaffold children's repeating patterning skills. A recent preschool pattern intervention illustrated how typical classroom practice could be modified to improve children's repeating patterning skills (Papic et al., 2011). Children drew repeating patterns from memory, identified missing items in patterns, identified errors in patterns, and created new patterns using different materials. They were prompted to identify similarities and differences within and between repeating patterns, and to identify the unit of repeat and the number of repetitions of the unit. These children had better repeating patterning skill at the end of preschool, and these benefits were maintained through the end of kindergarten. Patterning skills can also be promoted by encouraging preschool children to explain example patterns (Rittle-Johnson, Saylor, & Swygert, 2008), by providing children with explanations (Rittle-Johnson et al., 2015), and by using abstract labels to identify the unit of repeat (e.g., "The part that repeats in my pattern is A-B-B;" Fyfe, McNeil, & Rittle-Johnson, 2015). Overall, we should be promoting children's repeating patterning skills using evidence-based practices, not minimizing their importance.

The current findings, combined with previous longitudinal evidence (Lauer & Lourenco, 2016; Mix & Cheng, 2012; Verdine et al., 2017), also suggest that spatial skills should be considered within early math standards. Currently, spatial reasoning is only mentioned once in the Common Core State Standards (2010) for math, as part of a content standard for Kindergarten. Spatial skills seem to support math thinking and learning. However, in light of mixed experimental evidence that improving spatial skills also improves performance on math assessments (Cheng & Mix, 2013; Hawes et al., 2015; Lowrie et al., 2017; Xu & LeFevre, 2016),

we are more cautious in this recommendation. With this caution in mind, there are numerous promising and practical activities that seem to promote spatial skills. Children's experiences playing with blocks, puzzles, videogames and other spatial materials are thought to contribute to developing their spatial skills (Jirout & Newcombe, 2015; Levine et al., 2012; Newcombe, 2010; Verdine et al., 2014a; Verdine et al., 2014b). Scaffolding can enhance the impact of these experiences. In particular, teacher-guided block play in which children are asked to build structures with specific constraints and are prompted to modify their designs in specific ways supports better spatial skills in kindergarten children than unguided block play (Casey et al., 2008). Overall, the current evidence base justifies future research testing alternative interventions to support patterning and spatial skills in early childhood, and whether they also promote better math knowledge, including numeracy knowledge.

Limitations and Future Directions

The current findings support the potential importance of both repeating patterning and spatial skills as unique contributors to early math knowledge. Floor effects on some assessments, such as the research-based patterning assessment and visual-spatial WM, may have masked or weakened some relations. More broadly, additional research is needed on whether the current findings generalize to different math topics (e.g., number vs. geometry), other types of spatial skills (e.g., mental rotation) or patterning skills (e.g., growing patterns), and to a broader range of children receiving different types of math instruction.

Most importantly, more experimental training studies are needed before strong causal conclusions can be drawn. Longitudinal studies often overestimate the sustained impact of treatment (Bailey, Duncan, Watts, Clements, & Sarama, 2017). There is some experimental evidence that improving patterning or spatial skills can improve math achievement, reviewed

above, but additional research is needed to understand why each skill set might promote math performance and learning, and under what conditions, and using rigorous control conditions (Burgoyne et al., 2017). For example, does patterning promote attention to rules and relationships that are critical within math? Do spatial skills support visual representations of math problems that support problem solving? Do these relations vary with age or math topic?

In conclusion, preschool children's repeating patterning and spatial skills predicted their concurrent and future math achievement. Although these findings are correlational and not causal, they converge with other experimental evidence for a causal connection between the two skills and math achievement in elementary-school age children (Cheng & Mix, 2013; Kidd et al., 2013; Kidd et al., 2014; Lowrie et al., 2017). Both theories of early math development and early math educational standards should be expanded to incorporate a role for patterning and spatial skills.

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Table 1

Sample Characteristics

	Frequency	Percent (%)
Ethnicity		
African-American	34	46.6
Caucasian or White	31	42.5
Biracial/Mixed Race	3	4.1
Hispanic or Latino	3	4.1
Asian or Pacific Islander	2	2.7
Financial Assistance*		
None	32	43.8
Some	13	17.8
Full	26	35.6
Language(s)*		
Multiple languages	9	12.3
English only	64	87.7
Special Education Services		
Yes	6	8.1
No	67	91.9

**Notes*. Financial data were missing for two participants. Languages spoken at home included Kurdish, Nepali, Spanish, and Amharic.

Table 2

Descriptive Statistics for Items on Teacher-Based Patterning Assessment at Time 1

Item number, type, and pattern unit	Proportion correct (SD)	Item-total correlation	Item difficulty (SE)
9. Match AB	.57 (.50)	.46	-.60 (.26)
5. Missing ABB	.55 (.50)	.40	-.54 (.26)
3. Missing AB	.51 (.50)	.60	-.38 (.26)
1. What's next AB	.47 (.50)	.49	-.09 (.26)
2. What's next ABC	.42 (.50)	.34	-.04 (.26)
4. Missing ABC	.42 (.50)	.40	.08 (.26)
7. Extend AABB	.42 (.50)	.78	.14 (.26)
6. Extend AB	.40 (.49)	.77	.19 (.26)
10. Match ABBB	.40 (.49)	.42	.19 (.26)
8. Extend ABC	.25 (.44)	.60	.87 (.27)

Notes. Items are listed in order of observed difficulty, and item number indicates order in which item was given. Negative item difficulty values indicate less challenging items

Supplemental Materials

Table S1

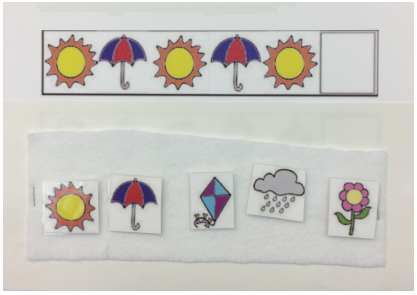
Hierarchical Multiple Regression Models: Spatial and Patterning Skills Predicting Numeracy Knowledge

	Numeracy at T1				Numeracy at T2				Numeracy at T2 with Num. at T1			
Variable	*B*(SE)	β	*p*	ΔR^2	*B*(SE)	β	*p*	ΔR^2	*B*(SE)	β	*p*	ΔR^2
Step 1				.44**				.43**				.47**
Age	.57(.59)	.10	.34		1.40(.55)	.27	.01*		1.25(.54)	.24	.02*	
Verbal Ability	.02(.01)	.20	.05†		.01(.01)	.11	.29		.01(.01)	.06	.59	
Verbal WM	.58(.12)	.50	.00**		.45(.12)	.42	.00**		.31(.13)	.29	.02*	
No. Knowledge T1	--	--	--		--	--	--		.25(.11)	.27	.03**	
Step 2				.05				.08*				.06†
Age	.44(.58)	.08	.45		1.28(.52)	.25	.02*		1.19(.52)	.23	.03*	
Verbal Ability	.02(.01)	.19	.06†		.01(.01)	.10	.31		.01(.01)	.06	.52	
Verbal WM	.44(.14)	.38	.00**		.29(.13)	.27	.02*		.21(.13)	.20	.12	
No. Knowledge TI	--	--	--		--	--	--		.18(.11)	.20	.11	
Visual-Spatial WM	.04(.08)	.06	.61		.07(.08)	.10	.37		.06(.08)	.09	.42	
Form Perception	.07(.04)	.18	.11		.02(.04)	.06	.57		.01(.04)	.03	.80	
Spatial Visualization	.05(.06)	.08	.42		.12(.05)	.23	.02*		.12(.05)	.22	.03*	
Step 3				.04				.06*				.05*
Age	.20(.58)	.03	.73		1.04(.51)	.20	.04*		1.01(.51)	.19	.05†	
Verbal Ability	.02(.01)	.18	.07†		.01(.01)	.10	.28		.01(.01)	.08	.39	
Verbal WM	.33(.14)	.29	.03*		.16(.13)	.15	.21		.13(.13)	.12	.34	
No. Knowledge T1	--	--	--		--	--	--		.10(.11)	.11	.35	
Visual-Spatial WM	.03(.08)	.04	.71		.06(.07)	.09	.43		.05(.07)	.08	.46	
Form Perception	.05(.04)	.14	.22		.01(.04)	.02	.81		.00(.04)	.01	.93	
Spatial Visualization	.03(.06)	.05	.60		.11(.05)	.21	.03*		.11(.05)	.21	.04	
Researcher-Pattern	.04(.09)	.05	.68		-.02(.08)	-.04	.76		-.03(.08)	-.04	.73	
Teacher-Pattern	.28(.15)	.22	.06†		.38(.13)	.33	.00**		.36(.13)	.30	.01**	

Notes. Age is age at time point of dependent variable. † $p < .1$. *$p < .05$. **$p < .01$.

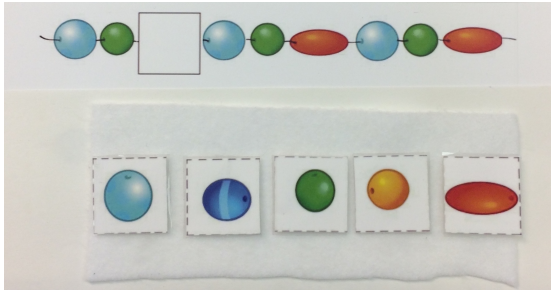
Figure 1. Sample Items on the Teacher-Based Patterning Assessment

What's Next Pattern AB



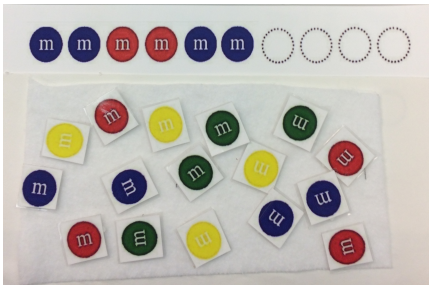
"What comes next in the pattern? Use one of these." [Experimenter gestures to response options.]

Missing Item Pattern ABC



"Find the missing bead [experimenter gestures to response options] to complete the pattern [experimenter gestures across pattern]."

Extend Pattern AABB



"Can you complete the pattern?" [Experimenter gestures to circles on the right of the pattern.]

Match Pattern ABBB

"Can you make the same kind of pattern using your pictures?" [Experimenter gestures to boxes below the model pattern.]

Table 3

Descriptive Statistics and Correlations for Key Variables

	Raw Score		Correlation											
Variable	*M*	*SD*	1	2	3	4	5	6	7	8	9	10	11	12
Control Variables														
1. Age at Time 1	4.5	0.3		.33*	.46*	.37*	.41*	.44*	.27*	.27*	.22	.32*	.38*	.46*
2. Verbal Ability	98.0	14.2			.42*	.37*	.27*	.36*	.31*	.22	.15	.22*	.39*	.36*
3. Verbal WM	1.5	1.5				.52*	.57*	.62*	.54*	.41*	.36*	.55*	.65*	.56*
Target Variables														
4. Research-Pattern	0.1	2.3					.56*	.88*	.44*	.48*	.42*	.57*	.57*	.46*
5. Teacher-Pattern	0.0	1.3				.35*		.89*	.41*	.40*	.34*	.48*	.64*	.65*
6. *Pattern Composite*	-0.0	0.9				.82*	.82*		.49*	.50*	.43*	.59*	.69*	.63*
7. Visual-spatial WM	2.6	2.3				.21	.14	.21		.51*	.38*	.78*	.50*	.53*
8. Form Perception	6.8	4.6				.33*	.20	.32*	.37*		.42*	.82*	.51*	.45*
9. Spatial Visualization	16.0	2.9				.30*	.16	.28*	.24	.31*		.77*	.43*	.43*
10. *Spatial Composite*	-0.0	0.7				.38*	.23	.37*	.69*	.77*	.74*		.61*	.59*
11. Math Time 1	-0.9	0.9				.33*	.42*	.46*	.22	.34*	.29*	.38*		.72*
12. Math Time 2	0.1	0.9				.18	.45*	.38*	.32*	.26*	.29*	.40*	.55*	

Notes. Values above the diagonal are raw correlations ($df = 70$). Values below the diagonal are partial correlations after controlling for age at Time 1, verbal ability and verbal WM ($df = 67$). *$p < .05$.

Table 4

Hierarchical Multiple Regression Models Predicting Math Knowledge

	Math at T1				Math at T2				Math at T2 with Math at T1			
Variable	*B*	β	*p*	ΔR^2	*B*	β	*p*	ΔR^2	*B*	β	*p*	ΔR^2
Step 1				.44**				.38**				.57**
Age	.24(.31)	.08	.44		.75(.34)	.24	.03*		.60(.28)	.20	.04**	
Verbal Ability	.01(.01)	.14	.19		.01(.01)	.11	.32		.00(.01)	.03	.74	
Verbal WM	.34(.07)	.56	.00**		.26(.07)	.41	.00**		.05(.07)	.09	.44	
Math Knowledge T1	--	--	--		--	--	--		.60(.11)	.58	.00**	
Step 2				.09*				.10**				.03
Age	.15(.29)	.05	.60		.67(.32)	.22	.04*		.59(.28)	.19	.04*	
Verbal Ability	.01(.01)	.12	.21		.01(.01)	.09	.40		.00(.01)	.02	.79	
Verbal WM	.24(.07)	.40	.00**		.13(.08)	.21	.09†		.01(.07)	.01	.94	
Math Knowledge TI	--	--	--		--	--	--		.52(.12)	.50	.00**	
Visual-Spatial WM	.03(.04)	.07	.52		.09(.05)	.22	.07†		.07(.04)	.18	.08†	
Form Perception	.04(.02)	.20	.06†		.02(.02)	.10	.36		.00(.02)	-.00	.99	
Spatial Visualization	.05(.03)	.15	.13		.05(.03)	.17	.10		.03(.03)	.10	.30	
Step 3				.07**[a]				.09**[a]				.04*[b]
Age	-.03(.28)	-.01	.93		.52(.30)	.17	.08†		.52(.27)	.17	.06†	
Verbal Ability	.01(.01)	.11	.24		.01(.01)	.10	.31		.00(.01)	.05	.56	
Verbal WM	.16(.07)	.26	.03*		.05(.08)	.07	.55		-.02(.07)	-.04	.75	
Math Knowledge T1	--	--	--		--	--	--		.43(.12)	.42	.00**	
Visual-Spatial WM	.02(.04)	.05	.66		.08(.04)	.20	.07†		.07(.04)	.18	.08†	
Form Perception	.03(.02)	.14	.16		.01(.02)	.07	.52		.00(.02)	.01	.94	
Spatial Visualization	.03(.03)	.11	.25		.05(.03)	.15	.12		.03(.03)	.11	.23	
Researcher-Pattern	.04(.04)	.10	.39		-.04(.05)	-.09	.44		-.05(.04)	-.13	.23	
Teacher-Pattern	.20(.07)	.30	.01**		.28(.08)	.40	.00**		.19(.07)	.28	.01*	

Notes. Standard errors are in parentheses. [a] $df = (8, 63)$. [b] $df = (9, 62)$. Age is age at time point of dependent variable. † $p < .1$. *$p < .05$. **$p < .01$.