What to Look for in Mathematics

2020 Colorado Academic Standards

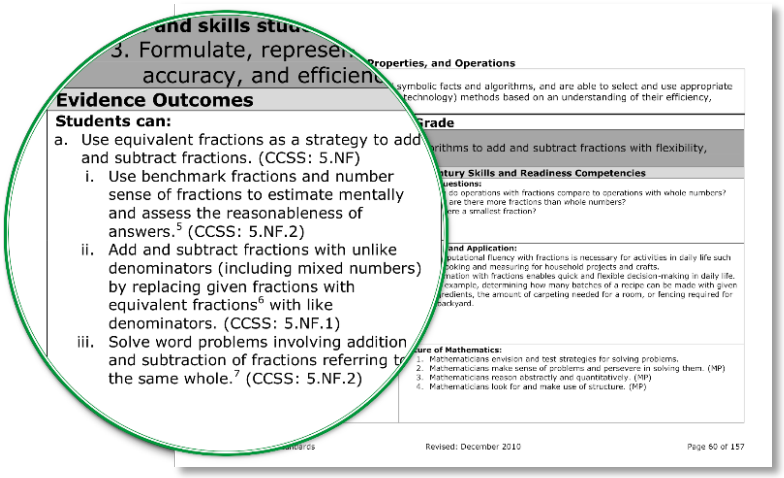
COLORADO
Department of Education

This document summarizes several changes and highlights to look for in the 2020 Colorado Academic Standards for mathematics.

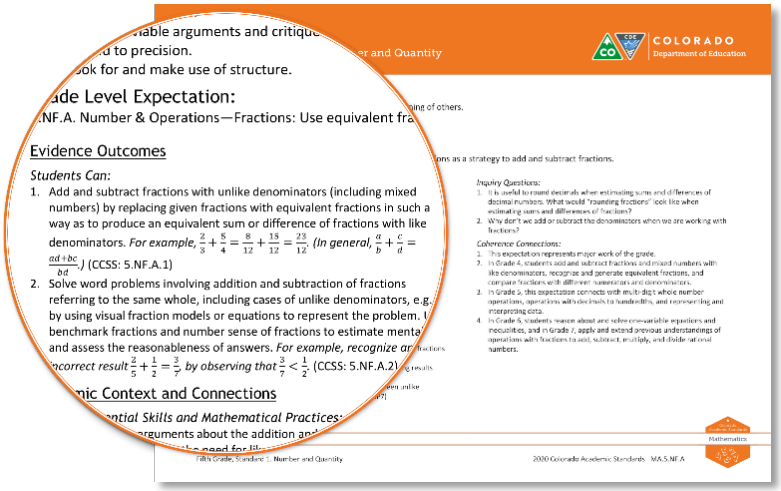
A Reorganization that Brings Greater Focus to Connected Mathematical Ideas

The 2010 mathematics standards often brought many evidence outcomes (EOs) together under a single grade level expectation (GLE). EOs were grouped using first-level EOs, multiple second-level EOs, and occasionally third-level EOs, and any level could contain footnotes.

Guided by public feedback, misalignment issues cited in a benchmarking report, and their own experience with the standards, the review and revision committee reorganized the standards into a greater number of GLEs, each more tightly focused on a mathematical idea with fewer EOs.



2010 Mathematics Standards



2020 Mathematics Standards

The committee combined some EOs and integrated any footnotes. In all but a small number of cases, the committee retained the language of individual EOs. These changes were made to add specificity and clarity, not to fundamentally change what students are expected to learn.

As an example, the first GLE under Standard 2 at high school in the 2010 standards had six first-level EOs, a total of 19 second-level EOs, and three third-level EOs. In addition, there were 11 footnotes used in these EOs, each containing an example of the mathematics described by an EO. The footnotes appeared four pages later in the text.

The mathematics review and revision committee reorganized this single 2010 GLE into six GLEs for the 2020 standards, one for each of the first-level EOs. Each first-level EO became a "new" GLE, and the sub-EOs from 2010 became first- and second-level EOs in 2020. The added benefit of these kinds of revisions across the standards is not only a tighter clustering of mathematical ideas, but each "new" GLE is supported by its own set of *Academic Context and Connections* (previously known as *21st Century Skills and Readiness Competencies*).

Where can I learn more?

Resource information

- For standards and instructional support in mathematics, see www.cde.state.co.us/comath/.
- For standards and instructional support in all subjects, see www.cde.state.co.us/standardsandinstruction/.

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Below is an annotated example of how larger 2010 GLEs have been reorganized into multiple 2020 GLEs. This type of reorganization is typical of the reorganization found across all grades and standards categories in the 2020 mathematics standards.

2010 – Fourth Grade

GLE: 1. The decimal number system to the hundredths place describes place value patterns and relationships that are repeated in large and small numbers and forms the foundation for efficient algorithms.

Evidence Outcomes / Students can:

a. Generalize place value understanding for multi-digit whole numbers. (CCSS: 4.NBT)
   i. Explain that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. (CCSS: 4.NBT.1)
   ii. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. (CCSS: 4.NBT.2)
   iii. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. (CCSS: 4.NBT.2)
   iv. Use place value understanding to round multi-digit whole numbers to any place. (CCSS: 4.NBT.3)

b. Use decimal notation to express fractions, and compare decimal fractions. (CCSS: 4.NF)
   i. Express a fraction with a denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.[1] (CCSS: 4.NF.5)
   ii. Use decimal notation for fractions with denominators 10 or 100.[2] (CCSS: 4.NF.6)
   iii. Compare two decimals to hundredths by reasoning about their size.[3] (CCSS: 4.NF.7)

Footnotes:

[1] For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.

[2] For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.

[3] Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of the comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

2020 – Fourth Grade

GLE: 4.NBT.A. Number & Operations in Base Ten: Generalize place value understanding for multi-digit whole numbers.

Evidence Outcomes / Students can:

1. Explain that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that* $700 \div 70 = 10$ *by applying concepts of place value and division.*
2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers base on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
3. Use place value understanding to round multi-digit whole numbers to any place.

GLE: 4.NF.C. Number & Operations – Fractions: Use decimal notation for fractions, and compare decimal fractions.

Evidence Outcomes / Students can:

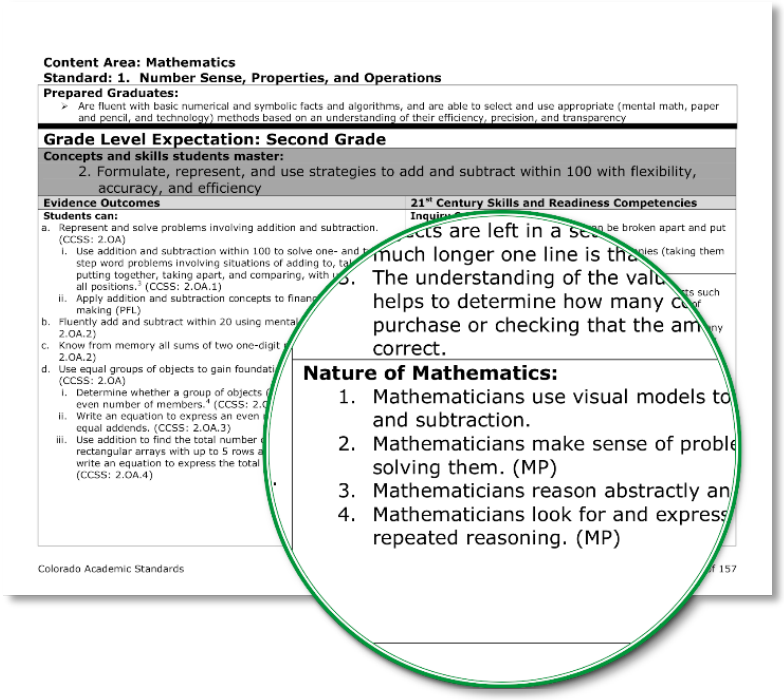
5. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. (Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators is not a requirement at this grade.) *For example, express* $\frac{3}{10}$ *as* $\frac{30}{100}$*, and add* $\frac{3}{10} + \frac{4}{100} = \frac{34}{100}$. (CCSS: 4.NF.C.5)
6. Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as* $\frac{62}{100}$*; describe a length as 0.62 on a number line diagram.* (CCSS: 4.NF.C.6)

Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model. (CCSS: 4.NF.C.7)

Promoted Practices and Prepared Graduates

The 2010 Colorado Academic Standards introduced the *Standards for Mathematical Practice* under the subheading *Nature of Mathematics*. Only the titles of the *Practices* were included, along with a simple "(MP)" citation. Since 2010, mathematics educators across Colorado have embraced the *Practices* and elevated their status to support the learning of mathematical content with the strategies and techniques of mathematicians.

The 2020 mathematics standards review and revision committee received pro-*Practices* feedback from the public and used their own experience to decide that the *Practices* should take a more prominent position in the revised standards.



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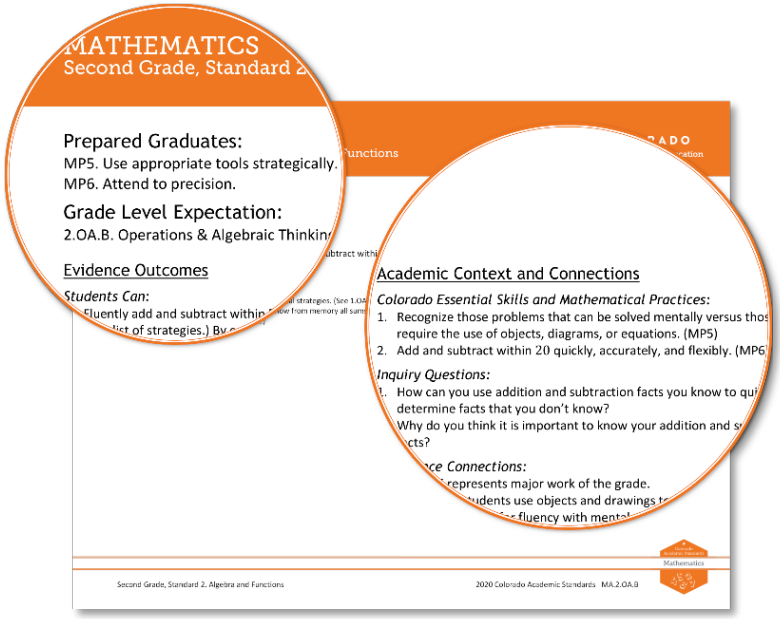

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In the 2020 standards, the *Standards for Mathematical Practice* appear as *Prepared Graduate* statements. These represent concepts and skills all graduates need to be successful after high school. Although students should have opportunities to engage in all eight Standards for Mathematical Practice when learning any mathematical content, the review and revision committee chose to align GLEs to only those practices for which there was the greatest opportunity to engage in each practice.

In addition, the committee replaced the *Nature of Mathematics* subheading with *Colorado Essential Skills and Mathematical Practices*. These are content-specific descriptions of each *Practice* named as a *Prepared Graduate* statement.

Inquiry Questions

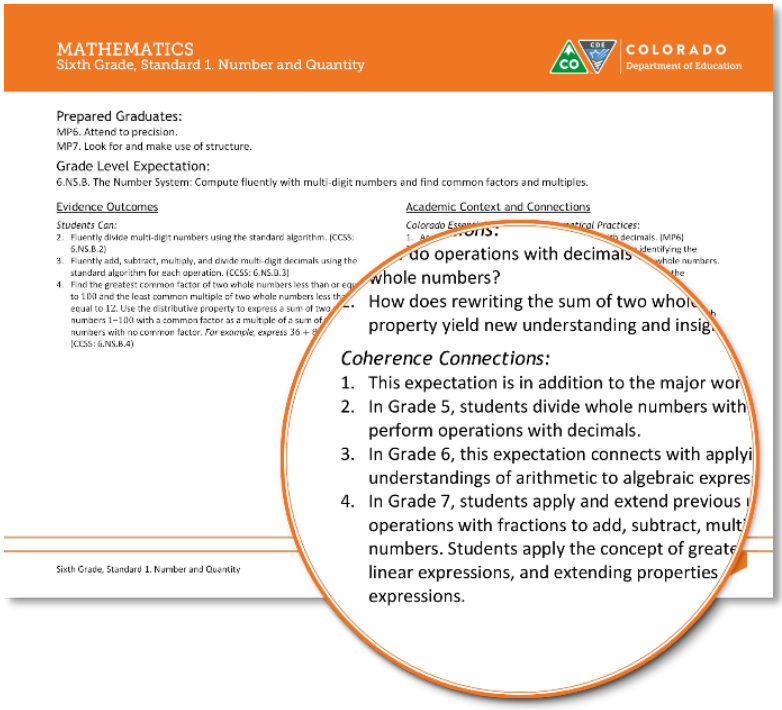
The review and revision committee kept the *Inquiry Questions* subheading from 2010 but made significant edits and additions. The committee designed each question to prompt deeper thinking about the mathematical content described by the grade level expectations and evidence outcomes.

Coherence Connections

The review and revision committee replaced the 2010 *Relevance and Application* subheading with *Coherence Connections*. This new subsection is designed to help teachers quickly connect mathematical ideas to other content in previous, current, and future grades.

Coherence Connections also indicates whether the mathematics described by a grade level expectation is *major*, *supporting*, or *additional* work of each grade.

In high school, *Coherence Connections* helps teachers interpret how grade level expectations and evidence outcomes fit within a coherent system of teaching and learning mathematics. This can include explaining the relevance and application of the mathematics, thus retaining some of those ideas from the 2010 standards.



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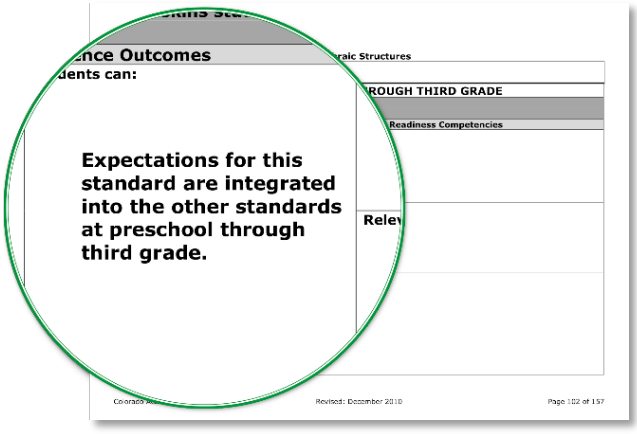


2010 Mathematics Standards

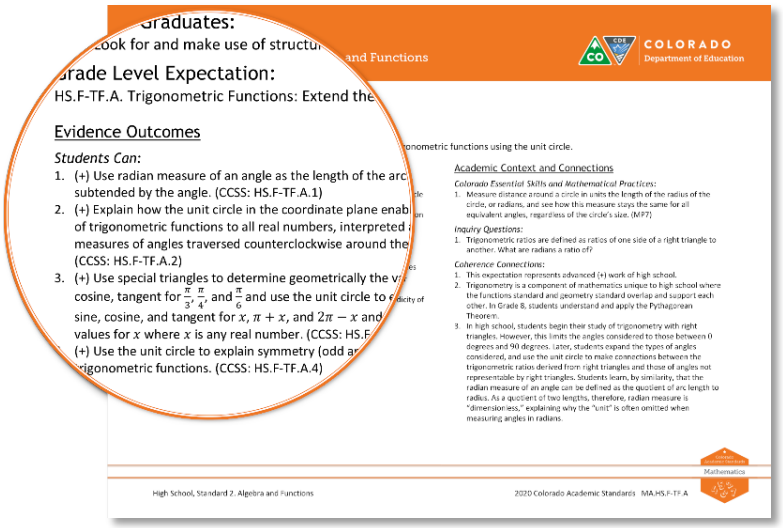
In other elementary grades, the review and revision committee shifted some content to fill standards categories that had previously been left empty. This shifted content represents the same content as was in the 2010 standards but is now located under a different category in the same grade. Most content focused on operations is now found under "Algebra and Functions" and measurement-focused content is now found under "Data, Statistics, and Probability."

Preschool and Early Elementary

The review and revision committee's preschool revisions are a significant change compared to the 2010 standards, but a minor adjustment when compared with CDE's 2013 *Early Learning Development Guidelines* and the 2015 *Head Start Early Learning Outcomes Framework*.

2020 Mathematics Standards

Advanced High School Standards



2020 Mathematics Standards

New in the 2020 high school standards are advanced evidence outcomes, designated with a (+) symbol. The review and revision committee included these outcomes to help support students in their last few semesters of high school who wish to prepare for future study in mathematics or other STEM fields.

In making this decision, the committee consulted public feedback from the 2010 standards, advanced standards in the *Common Core State Standards*, research about the relevance of mathematics for college and careers, guidance used by curriculum publishers, and testing frameworks from SAT and PARCC. For each evidence outcome identified as potentially advanced by public feedback and members of the committee, the committee checked these resources to ensure that any 2020 evidence outcomes marked as advanced (+) earned that designation. Below are those HS evidence outcomes that have been designated as advanced in addition to those already labeled (+) in the CCSS. (HS.A-SSE.B.4 was split into two evidence outcomes, with deriving the formula marked as advanced.)

Standard 1: Number and Quantity

(+) Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. (CCSS: HS.N-RN.A.3)

Standard 2: Algebra and Functions

Use the formula for the sum of a finite geometric series (when the common ratio is not 1) to solve problems. *For example, calculate mortgage payments.*★ (CCSS: HS.A-SSE.B.4)

a. (+) Derive the formula for the sum of a finite geometric series (when the common ratio is not 1). (CCSS: HS.A-SSE.B.4)

(+) Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.* (CCSS: HS.A-APR.C.4)

(+) Use radian measure of an angle as the length of the arc on the unit circle subtended by the angle. (CCSS: HS.F-TF.A.1)

(+) Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. (CCSS: HS.F-TF.A.2)

(+) Model periodic phenomena with trigonometric functions with specified amplitude, frequency, and midline.★ (CCSS: HS.F-TF.B.5)

(+) Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle. (CCSS: HS.F-TF.C.8)

Standard 3: Data, Statistics, and Probability

No additional evidence outcomes were identified as advanced.

Standard 4: Geometry

(+) Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. (CCSS: HS.G-CO.D.13)

(+) Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. (CCSS: HS.G-C.B.5)

(+) Derive the equation of a parabola given a focus and directrix. (CCSS: HS.G-GPE.A.2)